UiO : **Institutt for lærerutdanning og skoleforskning**
Det utdanningsvitenskapelige fakultet

Matematikkundervisning i Singapore

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My background

- B.Sc. University of Oslo
- Ph.D. University of California, Berkeley
- Dept. of Math, National Univ. of Singapore 1989 to 2011
- Vice-President of the Singapore Mathematical Society
- Consultant for textbooks for Ministry of Education in Singapore
- Chair of organizing committee for Singapore Mathematical Olympiad

My background 2

- Joint position with ILS (Dept. of Teacher Education) and Dept. of Math. at the Univ. of Oslo in 2011

Comparing mathematics education in Singapore and Norway

- Singapore and Norway are on opposite ends of the spectrum in many ways
- There are both good and bad things in Singapore. I want to present both.

Comparing mathematics education in Singapore and Norway 2

- Is Singapore's success due to the cultural context or to the way they teach mathematics?

- The answer is BOTH!

Myths about Singapore

- Many people in the West are willing to believe the most incredible things about Singapore
- I don't have time to debunk all of them, but let me just address two
- The suicide rate in Singapore is lower than in Norway
- They do not have 10 hours of math classes each week

Background about Singapore

Background about Singapore 2

- In Singapore, immigrant workers replaced the native Malays
- 75% Chinese, mostly Hokkiens from Fujian Province (same as Taiwan), but also Cantonese (same as Hong Kong), Teochew, Hakka and Hainanese
- 15% Malays who by law are required to be Muslims
- 10% Indians, mostly Tamils

Background about Singapore 2

Language most frequently spoken at home (%)			
Language	1990	2000	2010
English	18.8	23.0	32.3
Mandarin	23.7	35.0	35.6
(non-Mandarin) Chinese Dialects	39.6	23.8	14.3
Malay	14.3	14.1	12.2
Tamil	2.9	3.2	3.3

40 % of Indians speak other Indian languages, and pupils can choose Bengali, Gujarati, Hindi, Punjabi or Urdu at school

Only on third of the pupils use the language of instruction at home

Main difference between Singapore and Norway

- Singapore is a result oriented society.
- Norway is a process and ideology oriented society.
- Deng Xiaoping (邓小平) said "it doesn't matter whether a cat is black or white, if it catches mice it is a good cat"
- School politics in Singapore focuses on mice mortality.
- School politics in Norway focuses on the color of the cat.

School and politics

- In Singapore there is strong disagreement between the government and the opposition, but not when it comes to school politics.

Changes in education policies

- HENG Swee Keat was minister of education from 2011 to 2015 and implemented many changes
- The names of the top scorers at PSLE (Primary School Leaving Exam) are no longer released
- Instead the ministry releases stories about "underdogs" who have done well
- They have also stopped revealing the highest and lowest scores

Changes in education policies 2

- HENG's slogan was "Every school, a good school"
- Stopped annual ranking of secondary school
- Principals put good teachers to teach N(T)

Primary school

- Primary school 6 years (pass around books)
- After Primary 4, the weaker 10-15% are streamed into a different program called Foundation Maths
- There is an important exam (PSLE – Primary School Leaving Examination) at the end of Primary 6, when the kids are 12 years old, which determines which secondary school they will go on to

Secondary school

- Secondary school 4 (or 5 or 6) years with 3 courses
- 60% in Express course leading to O-level in four years
- Two thirds of these take Additional Mathematics course in Sec 3 and Sec 4

Secondary school 2

- 25% in Normal (Academic) course, which is similar to Express but at a slower rate
- After four years they take N(A)-level
- 75% do well enough at N(A)-level to be allowed to take an additional year N5 and take O-level after five years

Secondary school 3

- 15% in Normal (Technical) course leading to N(T)-level in four years
- N(T) is different from the other courses, and is targeted at weaker students
- 98% pass N(T)-level
- Those who do well, can continue on to N4, and take N(A)-level after five years, and if they do well there, they can continue on to N5 and take O-level after six years

Post-secondary

- Junior college 2 years (30%), Institute of Technical Education 1-2 years (20%), polytechnic 3 years (50%)
- JC (Junior college) corresponds to the academic track in Norwegian upper secondary school
- ITE (Institute of Technical Education) is purely vocational training
- Polytechnic is vocational training with theory

Post-secondary 2

- 25% of the cohort goes on to local universities, 20% to overseas universities
- Most JC students go on to university
- Many poly students go on to university
- ITE students rarely go on to university, but a handful do go from ITE to poly to university

Structure of the school system

- Singaporean kids start school in January of the year they turn 7, so they are half a year older than Norwegian kids

- The Singaporean system is 6+4+2, while the Norwegian is 7+3+3

- When I compare grades, I equate Norwegian and Singaporean grades for primary school and barneskole

Structure of the school system 2

- When I compare grades for Secondary school, I ignore Norwegian 7th grade (last year of primary school), and equate Singaporean Sec 1 through 4 with the three years of Norwegian lower secondary school and the first year of upper secondary school (grades 8 through 10 and VG1)
- I compare the two years of junior college with the two final years of upper secondary school (VG2 and VG3)

Social mobility through education 1

- Kids who perform well are given great opportunities, regardless of their social background
- They get lots of support from teachers and schools
- Many of the top students come from poor families
- There is less upward mobility among Malay and Indian students
- Malay girls do better than Malay boys

Social mobility through education 2

- Important exams like the PSLE at the age of 12 may favor students with educated parents

Dropout rates

- There is little social security or welfare in Singapore
- This makes education more important, both for the people and the government
- It is important for the government to minimize the dropout rates

Dropout rates 2

- They focus on making sure that the weaker students get some kind of qualification
- It is not seen as a problem if this is a lower qualification than what the regular students get
- It is possible for weaker students to make it to university, but it takes longer time than in Norway, and is most relevant for students in certain areas (electronics and biomedicine)

Overview
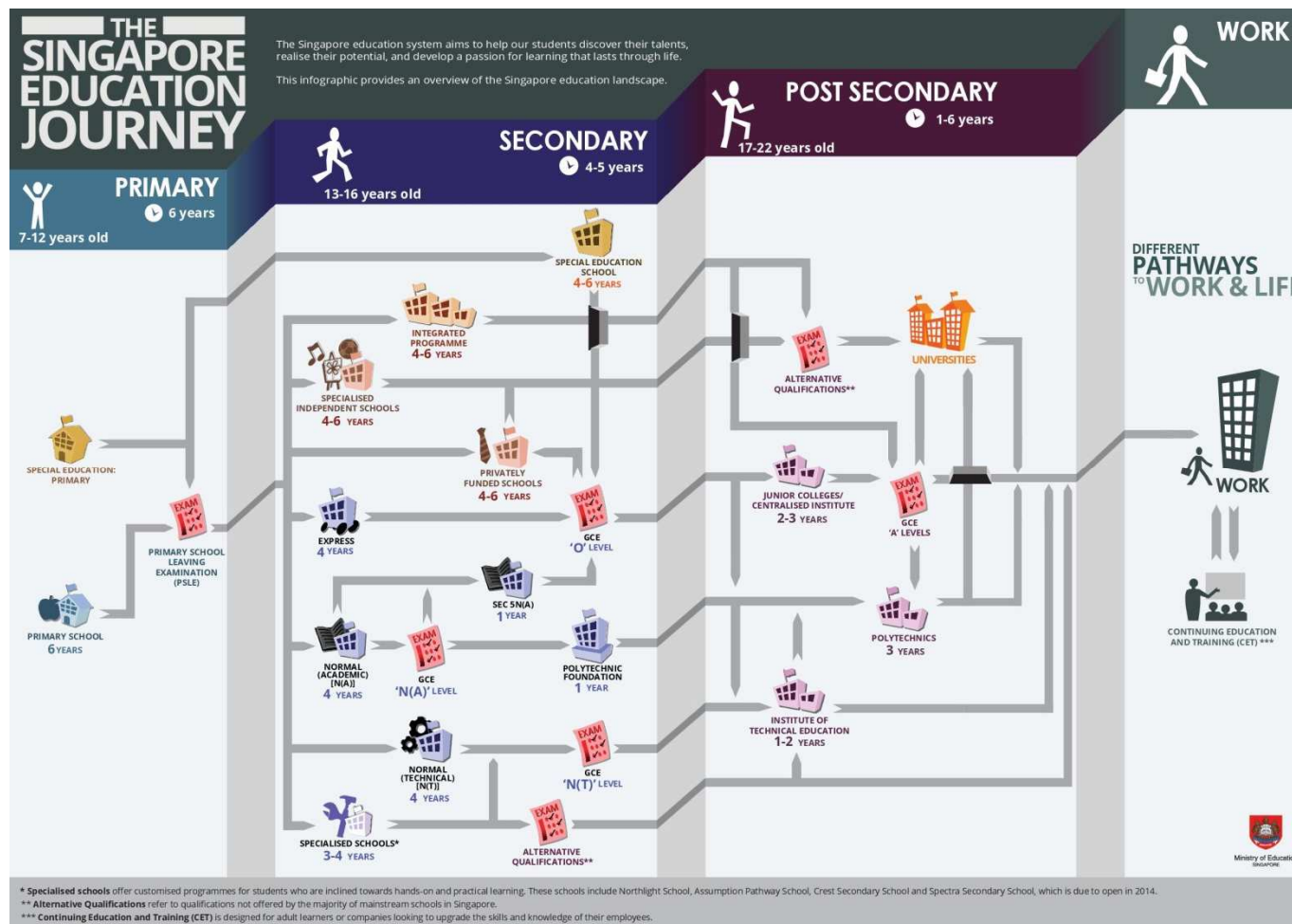
Scoring well at exams gives you celebrity status

PSLE = Primary School Leaving Examination

Does education help you find a good job?

- The Prime Minister has a degree in Math from Cambridge

- His teacher, Béla Bollobás, says that he was his best student ever

- When he topped the Math Tripos at Cambridge, there had never been a bigger gap between first and second

- The President has a Ph.D. in Math

- Singapore is a mathocracy!

The prime minister codes

Mark Zuckerberg
about a month ago

It was an honor to host Prime Minister Lee Hsien Loong of Singapore at Facebook HQ today.

The Prime Minister is one of the only world leaders who knows how to code. To recognize his technical skills, we gave the PM this artwork designed at Facebook etched with lines from a computer program he wrote. Coding can be useful -- no matter what your job is!

👍 410,146   💬 10,790   ➤ 15,366

In May last year, PM Lee shared on Facebook a Sudoku solver programme he created, adding that it was his way of championing science and technology,

Does education help you find a good spouse?

- High education makes males attractive to female partners
- When FHM Singapore published a list of "Ten most layable women", two of them had math degrees

Scholarships

Scholarships 2

The education system does not fit everybody

- I've had many great students in Singapore, but my favorite student was a guy who almost flunked his A-level, who barely made it into university and who got D in physics and C in math in his first semester

- He approached me about doing an undergraduate research project on astronomy

The education system does not fit everybody 2

- At first I was skeptical because of his grades, but then he told me that his favorite band was from Norway

- It was not Aha, but Theatre of Tragedy! (A Black Metal band)

- I managed to inspire him to do better at school, and he ended up getting a Ph.D. in the US and then did post docs at IAS in Princeton and ETH before becoming professor at Bern

The education system does not fit everybody 3

- He was able to succeed because he had a solid grounding in arithmetic and algebra
- Norwegian students often lack basic skills in arithmetic and algebra

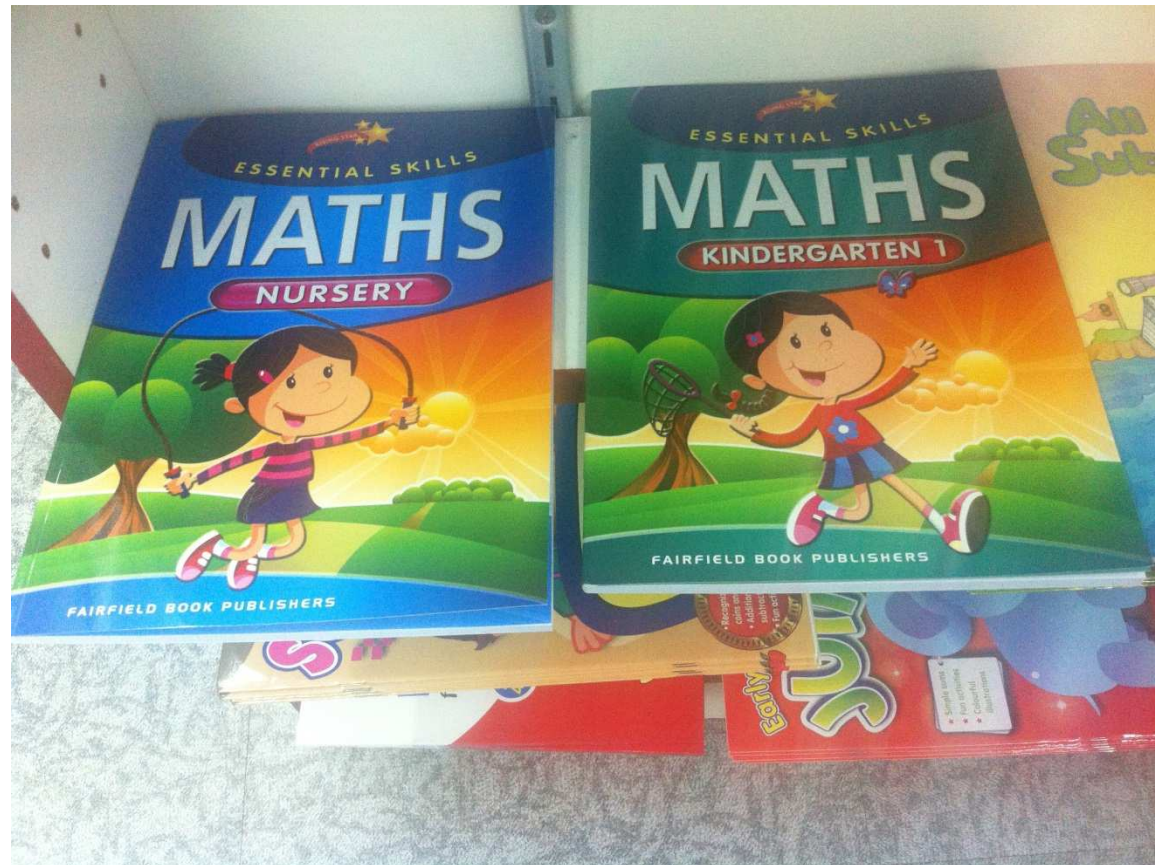# There is an incredible emphasis on education
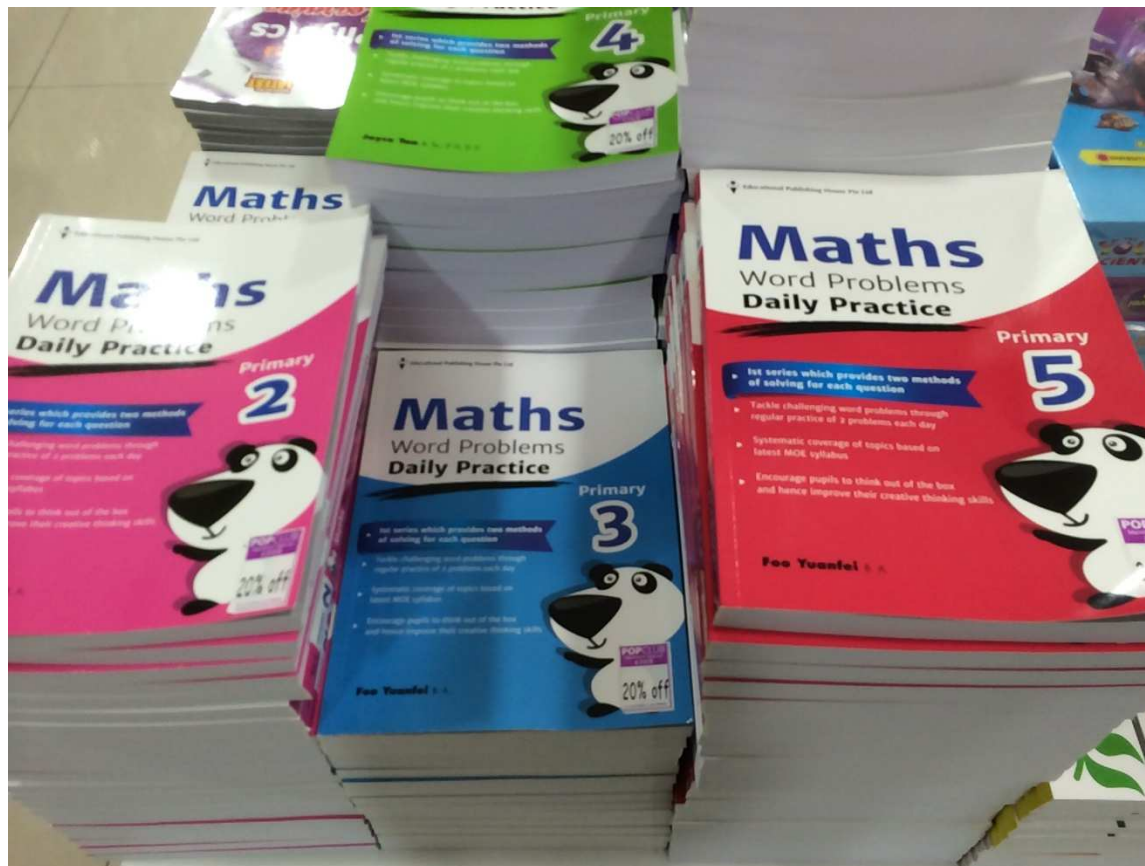
Bookstores are filled with math books

Bookstores are filled with math books 2

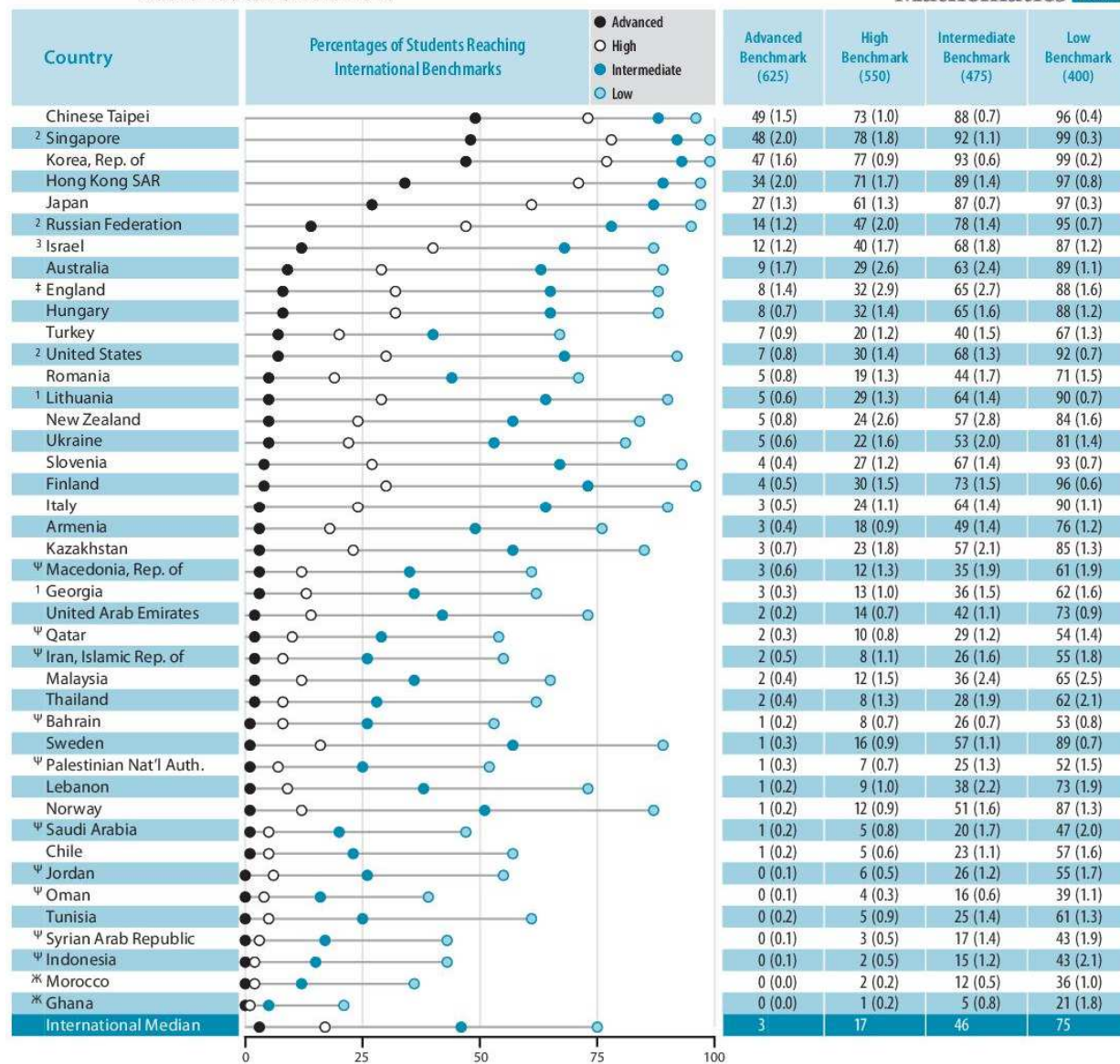# Bookstores are filled with math books 3

TIMSS

- There are four achivment levels in TIMSS: Advanced, high, intermediate, low
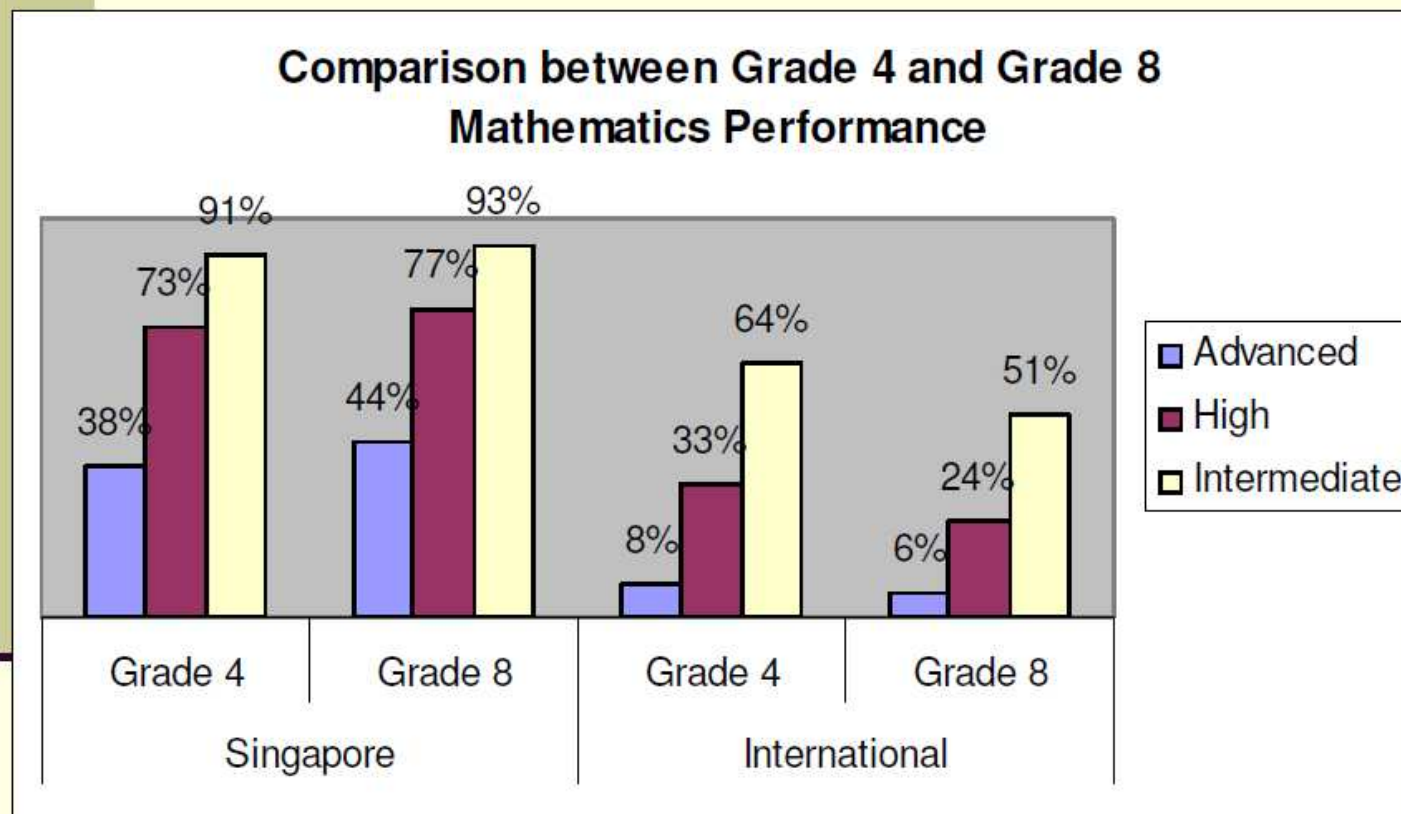
TIMSS 2

Exhibit 2.19: Performance at the International Benchmarks of Mathematics Achievement

TIMSS 2011 Mathematics 8th Grade

Percentages of Students Reaching International Benchmarks

● Advanced
○ High
● Intermediate
○ Low

Country	Advanced Benchmark (625)	High Benchmark (550)	Intermediate Benchmark (475)	Low Benchmark (400)
Chinese Taipei	49 (1.5)	73 (1.0)	88 (0.7)	96 (0.4)
² Singapore	48 (2.0)	78 (1.8)	92 (1.1)	99 (0.3)
Korea, Rep. of	47 (1.6)	77 (0.9)	93 (0.6)	99 (0.2)
Hong Kong SAR	34 (2.0)	71 (1.7)	89 (1.4)	97 (0.8)
Japan	27 (1.3)	61 (1.3)	87 (0.7)	97 (0.3)
² Russian Federation	14 (1.2)	47 (2.0)	78 (1.4)	95 (0.7)
³ Israel	12 (1.2)	40 (1.7)	68 (1.8)	87 (1.2)
Australia	9 (1.7)	29 (2.6)	63 (2.4)	89 (1.1)
‡ England	8 (1.4)	32 (2.9)	65 (2.7)	88 (1.6)
Hungary	8 (0.7)	32 (1.4)	65 (1.6)	88 (1.2)
Turkey	7 (0.9)	20 (1.2)	40 (1.5)	67 (1.3)
² United States	7 (0.8)	30 (1.4)	68 (1.3)	92 (0.7)
Romania	5 (0.8)	19 (1.3)	44 (1.7)	71 (1.5)
¹ Lithuania	5 (0.6)	29 (1.3)	64 (1.4)	90 (0.7)
New Zealand	5 (0.8)	24 (2.6)	57 (2.8)	84 (1.6)
Ukraine	5 (0.6)	22 (1.6)	53 (2.0)	81 (1.4)
Slovenia	4 (0.4)	27 (1.2)	67 (1.4)	93 (0.7)
Finland	4 (0.5)	30 (1.5)	73 (1.5)	96 (0.6)
Italy	3 (0.5)	24 (1.1)	64 (1.4)	90 (1.1)
Armenia	3 (0.4)	18 (0.9)	49 (1.4)	76 (1.2)
Kazakhstan	3 (0.7)	23 (1.8)	57 (2.1)	85 (1.3)
Ψ Macedonia, Rep. of	3 (0.6)	12 (1.3)	35 (1.9)	61 (1.9)
¹ Georgia	3 (0.3)	13 (1.0)	36 (1.5)	62 (1.6)
United Arab Emirates	2 (0.2)	14 (0.7)	42 (1.1)	73 (0.9)
Ψ Qatar	2 (0.3)	10 (0.8)	29 (1.2)	54 (1.4)
Ψ Iran, Islamic Rep. of	2 (0.5)	8 (1.1)	26 (1.6)	55 (1.8)
Malaysia	2 (0.4)	12 (1.5)	36 (2.4)	65 (2.5)
Thailand	2 (0.4)	8 (1.3)	28 (1.9)	62 (2.1)
Ψ Bahrain	1 (0.2)	8 (0.7)	26 (0.7)	53 (0.8)
Sweden	1 (0.3)	16 (0.9)	57 (1.1)	89 (0.7)
Ψ Palestinian Nat'l Auth.	1 (0.3)	7 (0.7)	25 (1.3)	52 (1.5)
Lebanon	1 (0.2)	9 (1.0)	38 (2.2)	73 (1.9)
Norway	1 (0.2)	12 (0.9)	51 (1.6)	87 (1.3)
Ψ Saudi Arabia	1 (0.2)	5 (0.8)	20 (1.7)	47 (2.0)
Chile	1 (0.2)	5 (0.6)	23 (1.1)	57 (1.6)
Ψ Jordan	0 (0.1)	6 (0.5)	26 (1.2)	55 (1.7)
Ψ Oman	0 (0.1)	4 (0.3)	16 (0.6)	39 (1.1)
Tunisia	0 (0.2)	5 (0.9)	25 (1.4)	61 (1.3)
Ψ Syrian Arab Republic	0 (0.1)	3 (0.5)	17 (1.4)	43 (1.9)
Ψ Indonesia	0 (0.1)	2 (0.5)	15 (1.2)	43 (2.1)
ж Morocco	0 (0.0)	2 (0.2)	12 (0.5)	36 (1.0)
ж Ghana	0 (0.0)	1 (0.2)	5 (0.8)	21 (1.8)
International Median	3	17	46	75

0   25   50   75   100

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Students are better at Grade 8!

Teaching in Singapore

- Model Method gives the students a problem solving tool before they have learned algebra
- Model Method makes it easier to learn algebra
- The syllabus is detailed, explicit and contains fewer topics
- Emphasis on explanations, theory, generalization and abstraction
- Focus on basic skill in arithmetic and algebra

Teaching in Singapore 2

- Good textbooks that have gone through a careful review process

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Syllabus

O Level Additional Maths Secondary Three/Four	
Topic/Sub-topics	Content
Calculus	
Differentiation and integration	Include: • derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point • derivative as rate of change • use of standard notations $f'(x)$, $f''(x)$, $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}[= \dfrac{d}{dx}(\dfrac{dy}{dx})]$ • derivatives of x^n, for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x, and $\ln x$, together with constant multiples, sums and differences • derivatives of composite functions • derivatives of products and quotients of functions • increasing and decreasing functions • stationary points (maximum and minimum turning points and stationary points of inflexion) • use of second derivative test to discriminate between maxima and minima • applying differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems • integration as the reverse of differentiation • integration of x^n for any rational n, $\sin x$, $\cos x$, $\sec^2 x$ and e^x, together with constant multiples, sums and differences • integration of $(ax+b)^n$ for any rational n, $\sin(ax+b)$, $\cos(ax+b)$ and $e^{(ax+b)}$ • definite integral as area under a curve • evaluation of definite integrals • finding the area of a region bounded by a curve and lines parallel to the coordinate axes • finding areas of regions below the x-axis • application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line with variable or constant acceleration Exclude: • differentiation of functions defined implicitly and parametrically • finding the area of a region between a curve and an oblique line, or between two curves • use of formulae for motion with constant acceleration

Norwegian syllabus

Læreplan i matematikk fellesfag   Læreplankode: MAT1-04

- vurdere forbruk og bruk av kredittkort og setje opp budsjett og rekneskap ved hjelp av rekneark
- undersøkje og vurdere ulike former for lån og sparing

Kompetansemål etter 1T-Y – Vg1 yrkesfaglege utdanningsprogram

Tal og algebra

Mål for opplæringa er at eleven skal kunne

- omforme ei praktisk problemstilling til ei likning, ein ulikskap eller eit likningssystem, løyse det matematiske problemet både med og utan digitale verktøy, presentere og grunngje løysinga og vurdere gyldigheitsområde og avgrensingar
- tolke, bearbeide, vurdere og drøfte det matematiske innhaldet i ulike tekstar
- vurdere, velje og bruke matematiske metodar og verktøy til å løyse problem frå ulike fag og samfunnsområde og reflektere over, vurdere og presentere løysingane på ein formålstenleg måte
- rekne med rotuttrykk, potensar med rasjonal eksponent og tal på standardform, bokstavuttrykk, formlar, parentesuttrykk og rasjonale og kvadratiske uttrykk med tal og bokstavar, faktorisere kvadratiske uttrykk, bruke kvadratsetningane og lage fullstendige kvadrat

Norwegian syllabus 2

Studiespesialiserende utdanningsprogram
Programområde for realfag

Funksjoner
Mål for opplæringen er at eleven skal kunne

- gjøre rede for begrepene grenseverdi, kontinuitet og deriverbarhet, og gi eksempler på funksjoner som ikke er kontinuerlige eller deriverbare
- bruke formler for den deriverte til potens-, eksponential- og logaritmefunksjoner, og derivere summer, differanser, produkter, kvotienter og sammensetninger av disse funksjonene
- bruke førstederiverte og andrederiverte til å drøfte forløpet til funksjoner og tolke de deriverte i modeller av praktiske situasjoner
- tegne grafer til funksjoner med og uten digitale hjelpemidler, og tolke grunnleggende egenskaper til en funksjon ved hjelp av grafen
- finne likningen for horisontale og vertikale asymptoter til rasjonale funksjoner og tegne asymptotene
- bruke vektorfunksjoner med parameterframstilling for en kurve i planet, tegne kurven og derivere vektorfunksjonen for å finne fart og akselerasjon

Kombinatorikk og sannsynlighet
Mål for opplæringen er at eleven skal kunne

- gjøre rede for begrepene uavhengighet og betinget sannsynlighet, og utlede og anvende Bayes' setning på to hendelser
- drøfte kombinatoriske problemer knyttet til ordnede utvalg med og uten tilbakelegging og uordnede utvalg uten tilbakelegging, og bruke dette til å utlede regler for beregning av sannsynlighet

Matematikk R2

Geometri
Mål for opplæringen er at eleven skal kunne

- utføre beregninger med tredimensjonale vektorer som er representert både geometrisk og på koordinatform
- bruke og tolke skalar- og vektorproduktet i beregning av avstander, vinkler, areal og volum
- bruke vektorregning til å finne liknings- og parameterframstillinger til linjer, plan og kuleflater
- beregne lengder, vinkler og arealer i legemer avgrenset av plan og kuleflater

Algebra
Mål for opplæringen er at eleven skal kunne

- finne og analysere rekursive og eksplisitte formler for tallmønstre med og uten digitale hjelpemidler, og gjennomføre og presentere enkle bevis knyttet til disse formlene
- gjennomføre og gjøre rede for induksjonsbevis
- summere endelige rekker med og uten digitale hjelpemidler, utlede og bruke formlene for summen av de *n* første leddene i aritmetiske og geometriske rekker, og bruke dette til å løse praktiske problemer
- regne med uendelige geometriske rekker med konstante og variable kvotienter, bestemme konvergensområdet for disse rekkene og presentere resultatene

The Model Method

Example (grade 3): The sum of two numbers is 36. The smaller number is one-third of the larger number. Find the two numbers.

larger   ?
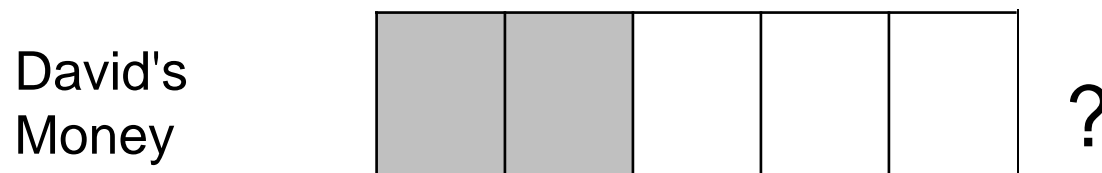
smaller   ?

36
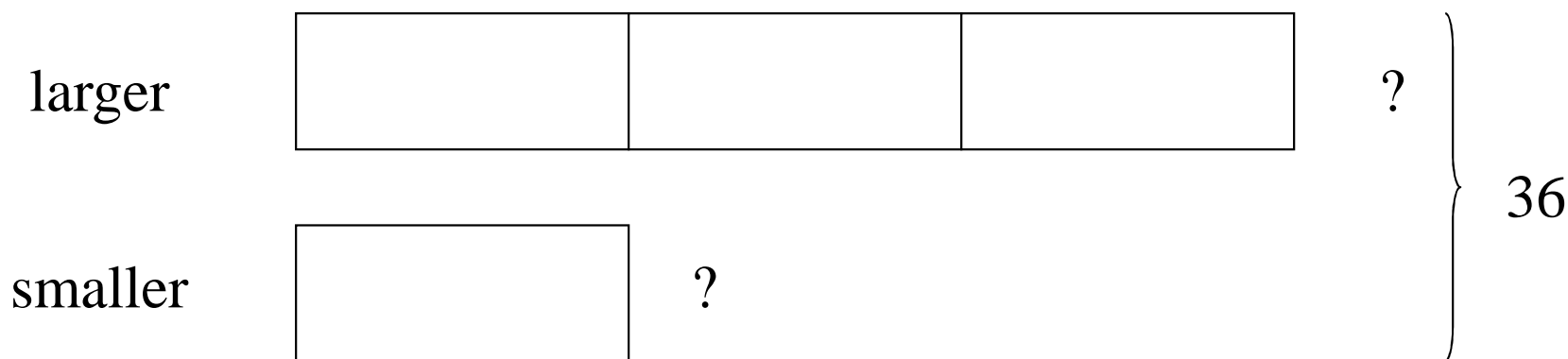
4 units = 36

1 unit = 9
3 units = 27

The numbers are 9 and 27

Example (grade 4): David spent 2/5 of his money on a storybook. The storybook cost $12. How much money did he have at first?

David's
Money

?

|-----12----|

2 units = 12
1 unit = 6
5 units = 30
David started with $30.

Primary 5

How would you solve this problem from the 5th grade Singapore textbook?

Mrs. Chen made some tarts. She sold 3/5 of them in the morning and ¼ of the remainder on the afternoon. If she sold 200 more tarts in the morning than in the afternoon, how many tarts did she make?

Primary 5

Mrs. Chen made some tarts.

Draw a bar to represent all the tarts.

Primary 5

She sold 3/5 of them in the morning…
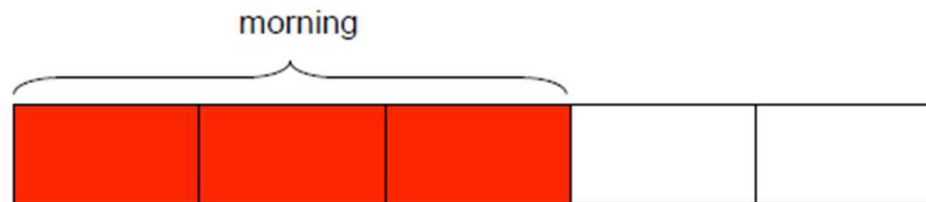
morning

Cut the bar into 5 equal pieces. Shade 3 of them to
show the tarts sold in the morning.

Primary 5

…and ¼ of the remainder in the afternoon.
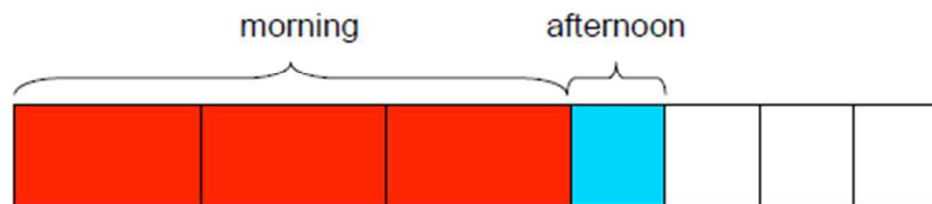
Cut the unshaded part into 4 equal pieces. Shade 1 of them to show the tarts sold in the afternoon.
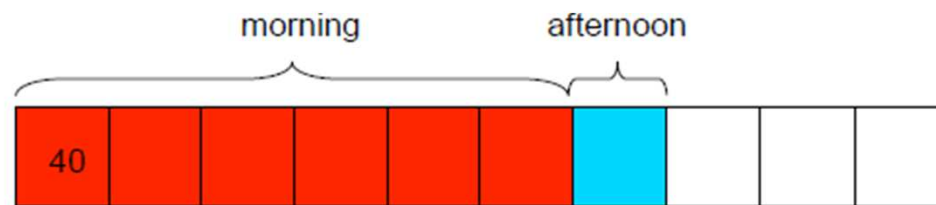
Primary 5

…and ¼ of the remainder in the afternoon.

morning   afternoon

Cut the red boxes in half to make all of the boxes
the same size.

Primary 5

…she sold 200 more tarts in the morning than in the afternoon…

There are 5 more morning boxes than afternoon boxes. We can use the unitary method to find the value of 1 box.

5 units = 200 tarts

1 unit = 200 ÷ 5 = 40 tarts

"Unitary" method

Primary 5

...how many tarts did she make?
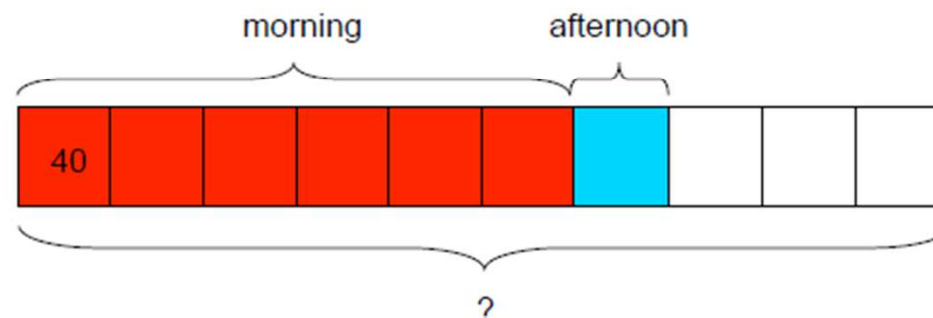
There are 10 boxes in all.

5 units = 200 tarts

1 unit = 200 ÷ 5 = 40 tarts

10 units = 40 x 10 = 400 tarts

Primary 5

Mr Tong bought a television set, an oven and a toaster.

The average cost of the oven and the toaster was $225 less than the average cost of the three items. The television set cost $980.

How much did Mr Tong pay for the three items altogether?

Average

- We can assume that the oven and the toaster cost the same, since we are only interested in their sum, which is twice the average.

Oven			

Toaster			

TV	225		

Average 2

Oven	
Toaster	
TV	225

Average of first two + 3x225 = 980
Average of first two = 305
Average of three = 530
Total cost = 1590

Teacher training in Singapore

- Secondary school and half of the Primary school teachers: University grad + one year at National Institute of Education

- Half of Primary school teachers: One year at National Institute of Education

- Overseas and local scholarships for students who are willing to serve bonds with Ministry of Education

Teachers in Singapore

- Teachers have high status and high salaries
- There are three career tracks – Leadership, Master Teachers and Specialist (curriculum, special needs etc)

Focus on teachers' content knowledge
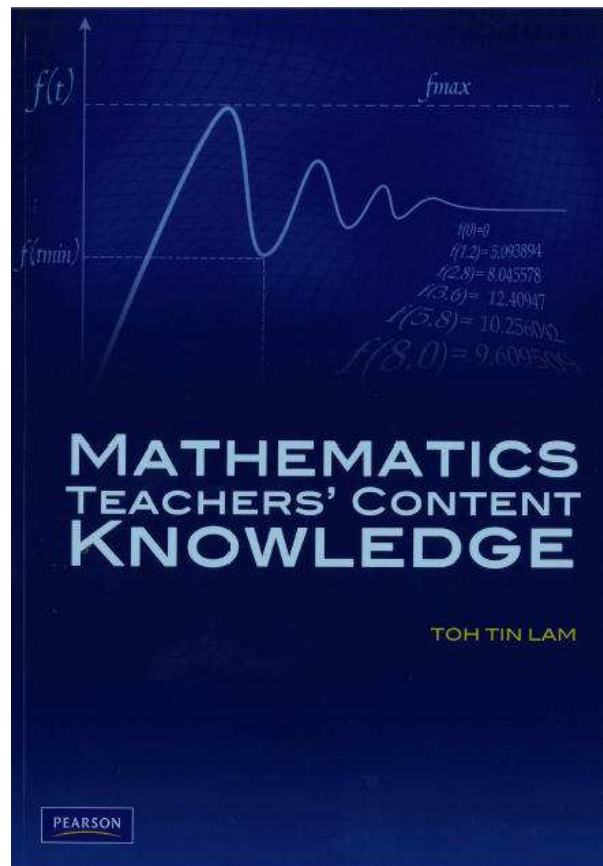
Singapore textbooks

- Singapore math textbooks have become a worldwide hit http://www.singaporemath.com/
- They are especially popular among home schoolers
- In the past they were published by the Ministry of Education (MoE), but now they have been privatized
- There is, however, a careful review by the MoE

How do Singaporean textbooks compare to Norwegian textbooks?

- In Norway there is no national review of textbooks

- There are many mistakes in the Norwegian textbooks, while I have never seen any errors in Singaporean textbooks

- In Norway, the publishing houses often have the final say when writing the textbooks

How do Singaporean textbooks compare to Norwegian textbooks? 2

- The Singapore textbooks emphasize understanding and enjoyment of mathematics

- Most Norwegian textbooks for primary and secondary school contain very few explanations, and focus instead on memorizing techniques learned from examples

Problem solving skills

- In Singapore there is a strong emphasize on problem solving
- In Norway there is hardly any discussion of problem solving skills in the textbooks (as demonstrated in a recent PhD thesis by Tom Kongelf)

Problem solving skills 2
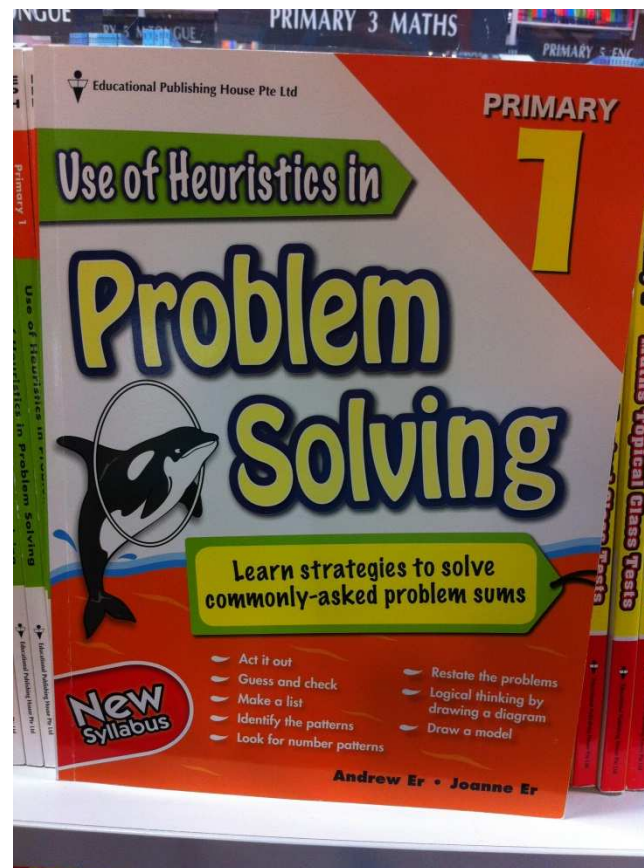
What can Norway learn from Singapore?

- Focus on basic skills in arithmetic and algebra
- "Huskeskole = Puggeskole", don't be afraid of explaining
- Understanding creates enjoyment of mathematics
- Less bureaucratic control of the syllabus
- Reintroduce review of textbooks by competent people, and not bureaucrats

What can Norway learn from Singapore? 2

- My wife was a teacher in a top-rated primary school in Singapore (Nan Hua Primary School) for 22 years. Part of her duties was to be "Discipline Mistress".

- This is not a job title in Norwegian schools

- After two years of applying, my wife got a job as an assistant in a Norwegian primary school

What can Norway learn from Singapore? 3

- Some of the teachers have a condescending attitude towards her and education in Singapore. They are very proud of the Norwegian school system and believe that education is Singapore is just mindless drilling and oppressive discipline.

- Some of them are gradually beginning to realize that my wife is an asset to them

References (available at Amazon)
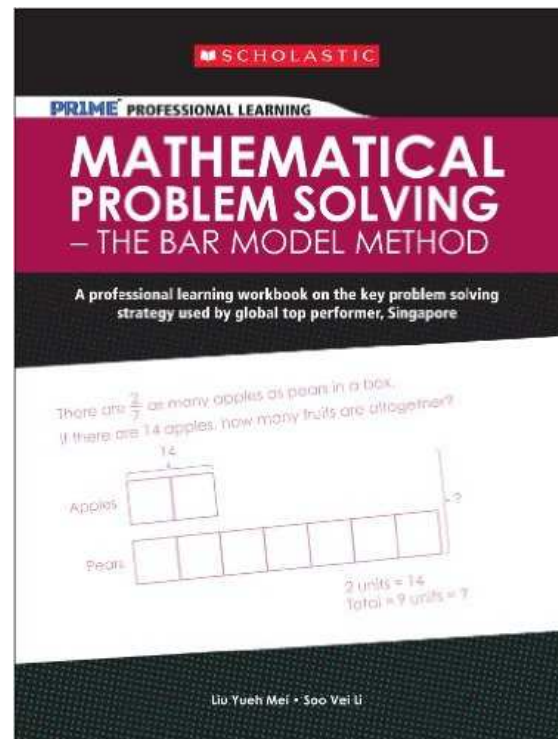
LIU & SOO - Mathematical Problem Solving -
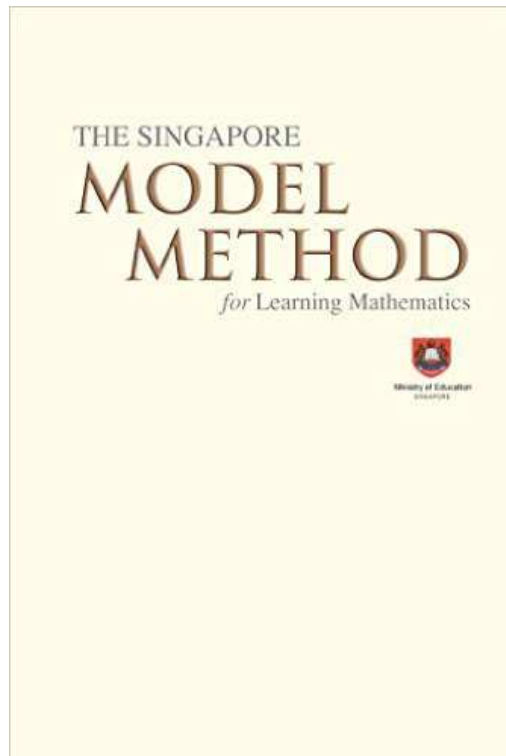The Bar Model Method
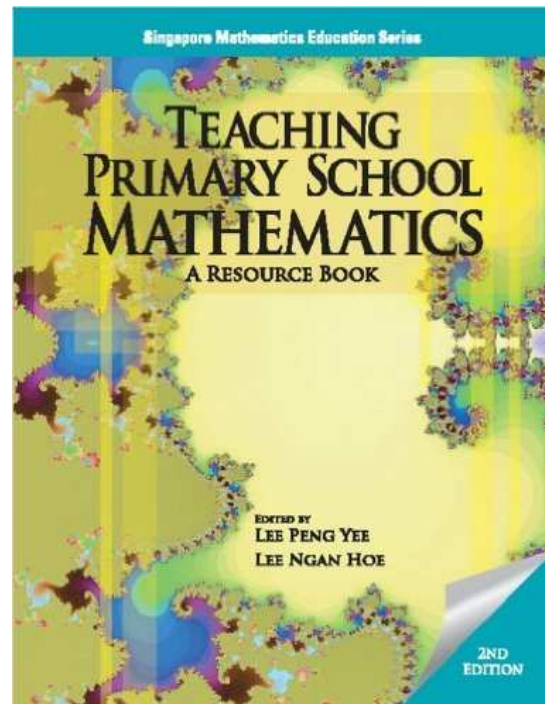Good overview

References 2
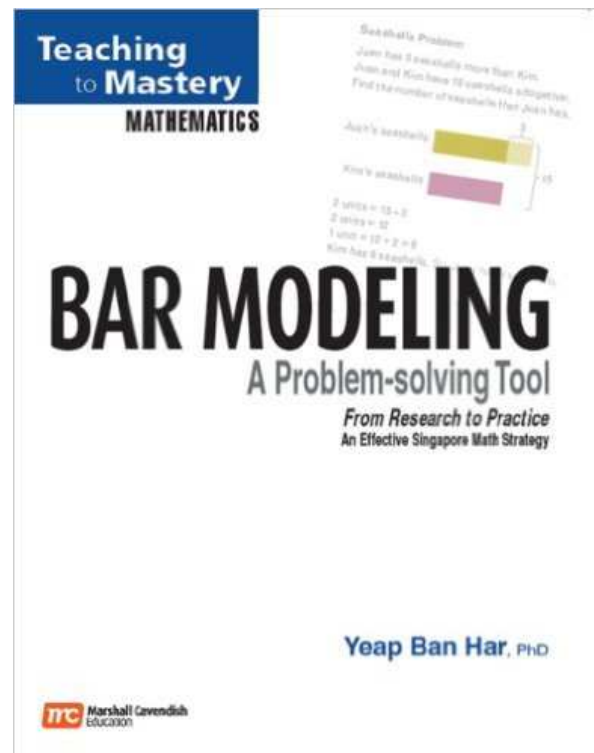
YEAP - Bar Modeling
Ban Har used to be at the Institute of Education, but is
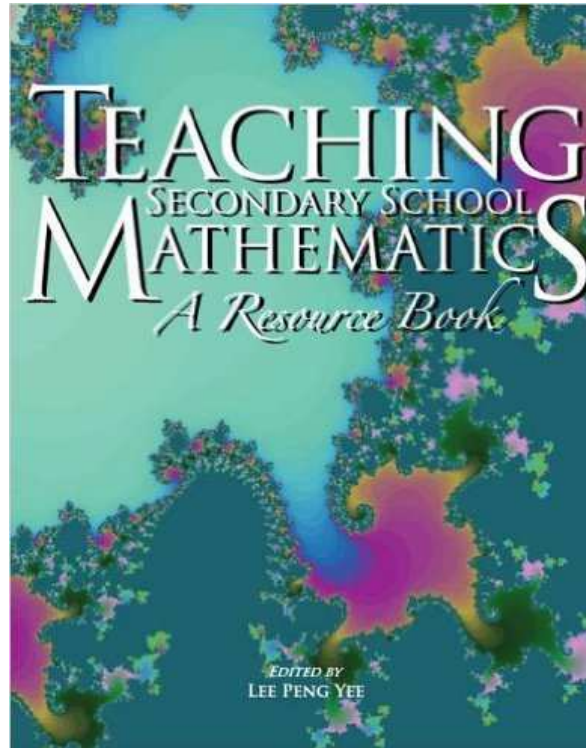now with a publisher

References 3

SEAH et al - The Singapore Model Method for
Learning Mathematics
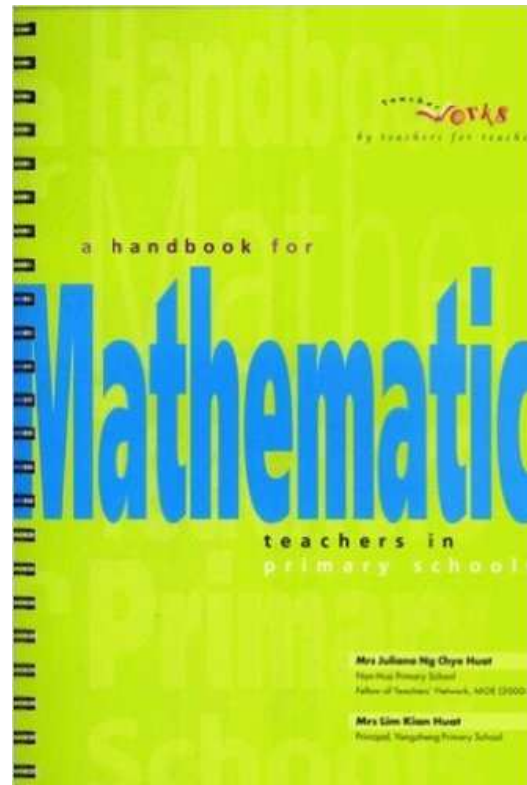The "official" book

References 4

LEE & LEE - Teaching Primary School Mathematics
Used in teacher education

References 5

LEE - Teaching Secondary School Mathematics
Used in teacher education

References 6

NG - A Handbook for Mathematics Teachers in Primary Schools
Juliana is a "Master Teacher"

Takk for meg!